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On the Frictional Resistance of Streaming Air

Über der Reibungswiderstand Stromender Luft

Ergob. Aerodyn. Versuchsanst. Göttingen, No. 3, 1927, P 1
Results of the Aerodynamic Research Station at Göttingen

Translated by

1. Theoretical Part

1. On the frictional resistance of streaming air

The frictional resistance of streaming air, which always attracts attention when air streams over a solid surface, is, owing to the inner mobility of the air, from another point of view like the "dry friction" of two solid bodies gliding over each other. The air layer which immediately borders the body adheres closely to the body and the actual frictional process therefore takes place entirely within the streaming air. Both the molecular motion and the turbulent mixing motion which exists throughout the streaming air, under certain conditions which frequently occur, cause a velocity interchange (austausch) between the adjacent layers and thereby an increasing retardation also of the outer layers from the adhering section. In the presence of this austausch the portion which is more rapidly approaching from the outside is arrested by mixing in the inner portions, and that which is approaching more slowly from the inside is driven on by that coming from the outside. This signifies, if one summarizes the dynamic effect of these motions by average values, a system of frictional tensions in the streaming air masses which is also imparted to the body in the stream, and in its total effect asserts itself as frictional resistance.

The regularity of the frictional resistance of air streaming over a surface can be related with the case for streaming in a long straight tube. Here the frictional tension on the wall (shearing stress = τ in the fluid in the immediate vicinity of the wall) is correlated with the pressure gradient in a simple manner. It is (equilibrium of a cylinder of radius r and length l)

$$2\pi r l \cdot \tau = \pi r^2 (p_1 - p_2) \text{ or } \tau = \frac{1}{2} \frac{p_1 - p_2}{l}$$

Through this relation the shearing stress τ is experimentally accessible according to the study of H. Blasius 1); for turbulent streaming one obtains, with u_c = velocity in the center for Reynold's numbers $\frac{u_c r}{\nu}$ from 1200 to about 50000, the approximate formula

$$\tau = 5 \frac{p_1 - p_2}{l} \left(\frac{r}{u_c} \right)^{1/4} \quad 1)$$

in which $\frac{1}{4}$ is a number approximately of the magnitude 0.045.2). For larger Reynold's numbers one would obtain smaller exponents instead of $\frac{1}{4}$ 3).

The frictional stress appears here dependent on the tube radius and on the velocity in the center (which probably can be derived by appropriate transformation of the average velocity which was found in the tube studies). One may suppose that neither the tube radius or the velocity in the center has an inward relation to the wall friction, however they appear here only formally, while in reality only the flow relations in the vicinity of the wall are of importance to the wall friction. On the other hand it can also be expected then that the velocity profile in the region of the wall is alone determined by the law of friction.

This train of thought has proved very productive. In order to express it in a formula one will look for the values which still remain when neither u , or r shall appear in the formula. Next let a velocity C be defined mathematically from the shearing stress τ in which one places $\tau = \frac{P}{2}$

Then a kind of Reynold's number can be formed right here from the kinematic viscosity ν , the wall interval y , and the velocity u . $Re = \frac{u y}{\nu}$

(One could also form Cy ; however formula 3) has proved to be suitable)
One can now fit the thoughts given above into formulas in which the formula

$$\tau = C \varphi(\beta) \quad (2)$$

is combined with equations 2) and 3). The various velocity distributions can be separated according to 4) only by the velocity factor $C = \frac{P}{2\tau}$ and by the measure y which was, corresponding to every u , changed as a consequence of 3). The form of the function φ is now obtained, if one assumes that equation 4) is applicable to the center of the tube, by introduction of the values of u and y , which are obtained from 2) and 4) for $\beta = \frac{P}{2\tau}$. One next puts down for 1) the general form

$$\tau = \frac{P}{2} F\left(\frac{P}{2\tau}\right) \quad (3)$$

so that due to 2) and 4)

$$\frac{P}{2} \left[\varphi(\beta) \right]^2 F(\beta)$$

thus

$$\varphi(\beta) = \frac{1}{\sqrt{F(\beta)}}$$

and consequently in general

$$\varphi(\beta) = \frac{1}{\sqrt{F(\beta)}} \quad (4)$$

From this relation we obtain u from 4) while from 3) we obtain $y = \frac{P}{2\tau}$. The relationship of u and y thus appears next in parametric form. For the special form of equation 1)

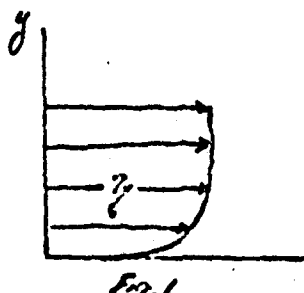
$$F(\beta) = \beta^{-\frac{1}{7}}; \quad \text{also} \quad \varphi(\beta) = \frac{1}{\sqrt{\beta^{-\frac{1}{7}}}} = \beta^{\frac{1}{14}}$$

Therefore

$$u = \sqrt{\frac{P}{2\tau}} \left(\frac{P}{2\tau} \right)^{\frac{1}{14}}$$

or after solving with respect to u :

$$u = \left(\frac{P}{2\tau} \right)^{\frac{1}{7}} \left(\frac{P}{2\tau} \right)^{\frac{1}{14}}$$



Thus we obtain u proportional to the 7th root of y (See Fig. 1 which illustrates this relation). In the center of the tube itself this law does not indeed hold true exactly, compared to our illustration the numerical factor of equation 6) still varies only a little through this deviation. In the case of Reynold's numbers under 50000 the proportionality with the 7th root of the wall interval throughout the study will be sufficiently good. In the case of higher Reynold's numbers, in which

the Blasius law is no longer exactly true, the velocity distributions also vary somewhat and indeed in the sense that instead of the 7th root of y there appears the 8th or 9th root.

The preceding relations for the tube flow can now also be employed for flow along a plate. In this case one must place the thickness δ of the section influenced by friction at the position of the tube radius. This section for the most part is relatively thin; it increases slowly along the stream. For the frictional resistance one now has two possibilities of expression: one the one hand it is the combined effect of all frictional stresses on the surface of the body, on the other hand it produces a retardation of the fluid mass which is affected by friction and can be calculated from its loss of impulsion. For uniformity in the broad sense the loss of impulsion can be obtained if u_1 signifies the velocity of the undistorted flow, and u signifies the velocity in the interval y from the wall.

$$J = \rho \int_0^\delta u(u_1 - u) dy$$

($\rho u dy$ = the mass flowing through in the unit of time between y and $y + dy$, $u_1 - u$ = velocity variation). If one accepts, according to what was developed earlier, that $u = u_1 \left(\frac{y}{\delta}\right)^{2/3}$ represents a good approximation, then one obtains a short equation

$$J = \frac{7}{72} \rho u_1^2 \delta \quad 7)$$

The calculation of resistance from the shear stress is made most convenient if one next ascertains the increase in resistance dW about the friction provoking surface around dl . For uniformity in the broad sense we again have

$$dW = \tau dl$$

or, if Blasius' Law 1) is used

$$dW = \frac{1}{2} \rho u_1^2 \left(\frac{N}{Re}\right)^{1/2} dl \quad 8)$$

On the other hand, according to equation 7), since $J=W$

$$dW = \frac{7}{72} \rho u_1^2 \frac{d\delta}{dl} dl \quad 9)$$

thus

$$\delta^{1/2} \frac{d\delta}{dl} = \frac{36}{7} \left(\frac{N}{Re}\right)^{1/2} l$$

$$\frac{1}{2} \delta^{1/2} = \frac{36}{7} \left(\frac{N}{Re}\right)^{1/2} l$$

or

$$\delta = \left(\frac{45}{7}\right)^2 \left(\frac{N}{Re}\right)^2 l^2 \quad 10)$$

and therefore equation 7) yields

$$W = \frac{7}{72} \left(\frac{45}{7}\right)^2 \rho u_1^2 l \left(\frac{N}{Re}\right)^2$$

$$W = 2.072 \frac{\rho u_1^2 l}{Re} \left(\frac{N}{Re}\right)^2 \quad 11)$$

Thereby we obtain the coefficient of frictional resistance c_f , if in addition the Reynold's number of the surface with length l , $Re = \frac{u_1 l}{\nu}$ is introduced,

$$c_f = 2.072 Re^{-1/2} \quad 12)$$

This law is surprisingly well fulfilled by the experimental results. It can only then naturally occur when the condition of disturbance appears in the immediate vicinity of the anterior edge. In the studies of Wieselsberger, in the 1st number, p. 121-126, we were informed that this was the case. The experimental points for the surfaces which are covered with six layers of spanning substance (plate numbers 151-154 of the first issue) were well reproduced in the case of the lower and middle Reynold's numbers by equation 12), in the

CASE of the highest Reynold's numbers, in comparison, there result deviations of a kind like those which appear opposite Blasius' law in the case of tubular resistance. The numerical value, 0.072 in equation 12), which is obtained from a not completely precise evaluation, will thus be able to lead to slight deviations. The above Wieselsberger studies are best demonstrated by the formula

$$c_f = 0.074 R^{-1/2} \quad (12a)$$

In the studies of Gebers (Footnote 1) and similar studies in which well sharpened plates were dragged through water, one still has no turbulence in the region next to the leading edge, but instead the flow here is laminar and the resistance much smaller than according to 12). The theory (Footnote 2), in the event that the flow along the whole plate is laminar, provides the formula

$$c_f = 1.327 R^{-1/2} \quad (13)$$

The latter is valid up to a critical value R' of $R = \frac{500,000}{\nu}$ (Footnote 3 - From here on out, ν which is a more usual term in resistance problems, will be used instead of the velocity z), which in the case of Geber's studies lies at about 500,000. For larger R laminar flow exists at the leading edge up to $l' = \frac{500,000 \nu}{v}$, from there on turbulent exists. One assumes that the friction in the turbulent portion is distributed according to the same law which we have derived at the outset; thus the deviation from the laminar basic results, in this section, in a lessening of resistance in the length l' , which corresponds to a lowering of c_f from the value of formula 12a), to that of formula 13), assuming both values for $R \approx R'$. In the value c_f for the entire plate, this lessening of resistance has a fractional ratio l'/l , for which R'/R can also be written. Thus for $R > R'$, $(c_f - c_{f_2}) R'/R$ is to be subtracted from the c_f value according to equation 12a), which, with $c_f - c_{f_2} = 0.0035$ and $R' = 485000$, gives the amount $1700/R$. Therefore, for turbulent frictional flow with laminar onset, one obtains the formula:

$$c_f = \frac{0.074}{R^{1/2}} - \frac{1700}{R^2}$$

(Footnote 4 - The verification of the section points of the c_f curves after equations 13) and 14) gives the exact values $R' = 487000$, $c_{f_2} = 0.00539$, $c_{f_2} = 0.00190$, and $c_{f_2} - c_{f_2} = 0.349$).

Instead of the value 17000, another numerical value can appear in a different case with smaller or larger critical Reynold's numbers (For example: when occasioned by larger or smaller initial turbulence).

Formula 14) agrees very very satisfactorily with the Gebers values so that the latter also provides a confirmation of the basic conception brought forward here. In Fig. 2 the three laws 12a), 13), and 14) are reproduced by the lines I, II, and III; thus the above mentioned measurements of Wieselsberger in air on surfaces of substances covered with 6 layers, those of Gebers on glass plates drawn through water, and moreover a study series made by Blasius with brass strips dragged through water, are the ones which cover the smaller Reynold's numbers. In their case a deduction of 0.0006 for each individual c_f value is made for the form of resistance, which improves the agreement and also is really justified.

The preceding considerations stem from the autumn of 1920. In the first issue (Vol. 1), it was only possible to add a short correction since the text in consideration was already printed. In this correction, our formula 14) is included with a slightly different numerical factor. All the questions which are dependent on the law of the 7th root have been taken up also by Prof. von Karman, Aachen. He had corrected this in the Zeitschrift für Angew. Math. und Mech., Vol 1 (1921): S 233 f. A valuable experimental contribution to the

question of frictional resistance of air flow meanwhile appeared in the dissertation of Van der Hegge Zijnen which was produced in the Burger's Laboratory in Delft (Thesis, Delft 1924), see also J. A. Burgers, Proc. of the 1st International Congress for Applied Mechanics, Delft, 1924, p 113 f. Additional literature is given there on p 128.

Figure 2

The relationships depicted here are based primarily on smooth plates also permit conclusions on the frictional resistance on airplane wings. The latter will in general always be somewhat larger than the resistance on smooth plates since the velocity in the case of airplane wings is unequally distributed, is greater at the place of larger velocity, and is not balanced by the lesser resistance at the place of smaller velocity. Nevertheless, a comparison is in order. It is proposed that the normal wing measurement (30m/sec velocity and 20cm wing depth) give a Reynold's number of about 420000 which is a value for which the sharpened plates yields directly a much smaller resistance. It is thus to be expected that wing measurements, in the case of an approximately three to four fold higher index value as are described for example on p 54 - p 62 of the first issue, are really reliable in relation to the frictional resistance. New studies on the profile resistance of the wings, depending on the index value, are found in this issue under number III, 7.

Over rough surfaces, the following observations could suffice: the deceleration of flow is here stronger because the frictional resistance is higher throughout than with even surfaces. To be sure, slightly rough surfaces could, especially in the case of small Reynold's numbers, exhibit just as technically smooth a flow if the roughness remains completely embedded in the laminar flowing layers. With higher Reynold's numbers, where this layer is thinner, such roughness could become noticeable due to the increased resistance. The coefficient of resistance of very rough surfaces is in general completely independent of the Reynold's number; see Issue 1, Fig. 75, p 123 (Surface material with filament peeled off). We are dealing with a smooth kind of body resistance; in contrast the coefficient of resistance is dependent on the ratio of size of curve (surface imperfection) to the object chord (size), such as also is obtained from the studies on channels with rough walls (See Hopf and Fromm, Zeitschrift fur angew. Math. und Mech., 3 (1923):329 and 339). It would also be possible to draw conclusions from the channel studies on the frictional resistance on plates, yet a remark on this matter may be omitted here until the laws for resistance of rough plates are better studied.

2. On eddy turbulence and its prevention.

Eddies which are formed by circulation behind the bodies are of greatest importance for the occurrence of resistance which results from the pressure differences. The origin of such turbulence is closely correlated with the fluid layer which is retarded by friction which surrounds the non flowing body like a garment. This frictional section is subjected to the same accelerating and retarding pressure differences which affect the undisturbed flow outside. If the outer flow accelerates due to a pressure drop in its direction of motion, then the retarded sections near the wall undergo an impulse in the direction in which they are already moving; thus the flow will continue its path along the surface of the body. If, on the other hand, a pressure drop against the direction of flow were to arise, then the free flow would be retarded by it. The sections next to the wall

moreover have a smaller velocity; indeed due to the friction from the outer flow they are dragged forward, checked however by the opposing pressure drop. If this be sufficiently strong then since their kinetic energy is sufficiently absorbed a reversal ensues and they now flow back against the flow which is departing outside (See Fig. 3). As newly retarded material always experiences the same fate, accumulations of the fluid arise and are mixed by the friction in rotation; these quickly form as eddies throughout the whole stream in a way that the pressure step which induced the formation of the eddy disappears or at least becomes very slight. It is mostly in this manner that the fluid stream is lost from the surface of the body and a more or less "deadspace" is left behind the body. The motion picture stills of Fig. 6 show the origin of the detachment from a circular cylinder.

The frictional processes play a somewhat different role in the case of a transversely accelerated flow, as for example what which occurs in curved tubes. Here a pressure drop across the direction of flow arises (for the subjection of the "centrifugal force"), the deviation of the individual fluid portions in this kind of a pressure drop however is different depending on their velocity; the retarded friction section is therefore more strongly retarded than the principal flow next to it, it flows therefore along the walls toward the inner side of the curvature. This process thus produces accumulation of slower fluid on the inner side of the curvature and though it also often results in no elimination it still represents a source of increased frictional loss. Since the original friction section will be laterally carried away a new one must always be formed again at its position.

In the foregoing discussion the pressure drop is considered as a force field which arises independently from the fluid flow. For the consideration of the partial processes this is probably permissible, however the pressure distribution itself is exactly assumed again as a product of the flow processes; this mutual connection makes the mathematical treatment of the hydrodynamic problems so changed and effected that one primarily controls mathematically only such problems, as with potential flow, in which it is possible to replace the dynamic relations by purely geometric ones. In other problems one must continue to be guided by experiment when more than qualitative information is desired.

The question about the means for avoiding formation of eddies and eliminating flow nevertheless belongs to those to whom our conception is capable of giving qualitatively valuable cues.

The simplest is the use of sufficiently slender forms as generally are the bodies of air ships, airplanes wings etc. Here the pressure increases in the direction of flow are not very large so that the drag action of the outer flow on the inner suffices for verifying it in front of the return flow. One shows that this "healthy flow" only occurs with airplane wings when the angle of attack is not too large, otherwise "the flow breaks away" which means in our view that the pressure increase of the "healthy" flow from the suction side becomes so large that back flows appear in the friction section and therefore that the entire flow is changed, since now they no longer lie close to the suction side, but instead a dead space or eddy space occurs between it and the wing. Similarly this holds true for an airship with too blunt a rear end.

The following observation is still important: If, as is the case with small index numbers (small Reynold's numbers) the frictional section flows lamina-ly, in general then it is very difficult to bring adjacent a flow in the face of a pressure

in the face of a pressure increase, since under these conditions the braking action of the outer flow is extraordinarily smooth on the friction section. If, on the other hand, the friction section is itself turbulent, then a much larger braking action exists and the subjection of the pressure increase is now much more possible. The airplane wing and models of such (as previously described) are always in this last mentioned condition. However, it is important to know that one gets large deviations in relation to the separation of flow if one comes downward below the limit under which the friction section remains laminar. With airplane wings this limit is essentially lower than with even plates (about $1/4$ of it, varying with the degree of turbulence of the air stream under study). The sudden fall of the resistance number (coefficient of resistance) of cylinders, spheres, and ellipsoids (See Issue 2, Figs. 24, 26, 27) likewise is correlated with the formation of turbulence of the friction layer. By means of the decreasing effect of drag, the point of separation moves further to the rear from a point somewhat in front of the middle of the sphere etc, through which the eddy field is materially reduced and the resistance reduced.

A rough surface increases the retardation of the friction layer; therefore the influence of the drag action is lowered, the separation point moves again more forward, and the resistance of the sphere etc, is again increased.

There are various means to prevent the formation of an eddy in the case of blunt shaped bodies which to be sure can not always be used. An especially effective means persists in that one can follow the body surface in the direction of flow with the air so that it nowhere runs more slowly than the flow. In this case nowhere does a retarding of the flow occur and consequently no eddy can develop. The study with two cylinders (See Fig. 4) which are counter-rotating has completely verified this.

If one assumes a single rotating cylinder, then with sufficient rotational velocity one of the two eddies will be completely suppressed; namely the one on the side where the wall motion follows in the direction of the flow, whereas the other eddy is fully developed. The end result of these processes, which is illustrated by Fig. 7, is a flow around the cylinder, similar to that in Fig. 5, which now also remains supported since each wall element now moves in the direction of flow. A very strong cross "force" combines with the flow as in Fig. 5 (The coefficient of cross force is thus theoretically $\sqrt{2} = 1.414$; in practice it is somewhat smaller). With lesser circumferential velocities, the flow, according to Fig. 5, in which the greatest velocity (above the cylinder) is four times the entering velocity, would precede the wall, and then we again would get eddy formation, the cross force will be less. With still larger circumferential velocities, a circular fluid ring appears to be around the rotating cylinder; the cross force moreover, if also not very large, exceeds the aforementioned amount.

Another method frequently used for the prevention of eddy formation consists in one removing the friction layer at the places where it would cause accumulation of material. ((When the situation is steady, this accounts for elimination of eddies by exhaustion behind the inner bodies)). The study shows that one actually prevents eddy formation by this means, and thus one can use considerably more blunt forms than would otherwise be possible. The work involved in this exhaustion is not large since we are dealing with relatively small amounts which must be suctioned off.

One may also try to influence (eddy formation) by emitting air streams in the direction of flow, in the delayed layers along the surface of the body. Since we are concerned here with a so-called (concentration) strengthening of the braking action, a favorable action must also be established here. The studies show, however, that the amounts of energy to be used for a satisfactory result must be much higher than with the exhaustion method. Until now, from different study arrangements of this kind, only the "jet wing" or "slotted wing" of Handley-Page-Lachmann have won practical significance. The air jet which flows through the air stream siphons off the friction layer of the forewing, and therefore makes it safer for the hind wing. At the latter a new friction layer is formed, which first of all is thin, and which, under the braking effect of the jets coming from the slot, is driven sufficiently forward. In this manner one can obtain a negative pressure on the upper side of the hind wing which is at least as large as that of a customary wing, and the forewing, whose hind margin is in the zone of the lowest negative pressure, can thus tolerate a still really lower negative pressure without interrupting the flow. With several slots the possible negative pressures of the forewings are still larger, still the energy expenditure also increases for the air streams which become important in the resistance of the system.